Section 8.5 Solutions

Section 8.5: systems on non-linear equations

Solve the following systems of equations.

1) $4x + y = 2$
$x^3 - 2 + y = 0$

Solve top equation for y

$4x + y = 2$

$y = -4x + 2$

Replace y in bottom equation with $-4x+2$

$x^3 - 2 + (-4x+2) = 0$

$x^3 - 2 - 4x + 2 = 0$

$x^3 - 4x = 0$

$x(x^2 - 4) = 0$

$x(x+2)(x-2) = 0$

$x = 0$ $\quad x+2=0$ $\quad x-2=0$

$x = -2$ $\quad x = 2$

Find a y to pair with each x

$x = 0$ $\quad y = -4(0) + 2$ $\quad (0, 2)$

$y = 2$

$x = -2$ $\quad y = -4(-2) + 2$ $\quad (-2, 10)$

$y = 10$

$x = 2$ $\quad y = -4(2) + 2$ $\quad (2, -6)$

$y = -6$

ANSWERS

$(0,2)$ $(-2,10)$ $(2,-6)$

3) $-2x+y=-5$
$x^2+y^2=25$

Solve top EQ. for y

$-2x+y=-5$

$y=2x-5$

Substitute into bottom EQUATION

$x^2+(2x-5)^2=25$

$x^2+(2x-5)(2x-5)=25$

$x^2+4x^2-20x+25=25$

$5x^2-20x+25=25$

$-25 \quad -25$

$5x^2-20x=0$

$5x(x-4)=0$

$5x=0 \qquad x-4=0$

$x=0/5 \qquad x=4$

$x=0$

Find a y to pair with each x

$X=0 \qquad y=2x-5 \qquad (0,-5)$

$y=2(0)-5$

$y=-5$

$X=4 \qquad y=2(4)-5 \qquad (4,3)$

$y=3$

ANSWER

$(0,-5)\ (4,3)$

5) $x^2 + y = 0$
$8x^2 - 4x - y = 0$

Solve top eq. for y

$x^2 + y = 0$

$y = -x^2$

Substitute into bottom eq.

$8x^2 - 4x - (-x^2) = 0$

$8x^2 - 4x + x^2 = 0$

$9x^2 - 4x = 0$

$x(9x - 4) = 0$

$x = 0$ $\quad 9x - 4 = 0$

$9x = 4$

$x = 4/9$

Find a y for each x

$x = 0$ $\quad y = -(0^2)$

$y = -0$ $\quad (0, 0)$

$y = 0$

$x = 4/9$ $\quad y = -\left(\frac{4}{9}\right)^2$ $\quad \left(\frac{4}{9}, -\frac{16}{81}\right)$

$y = -\frac{16}{81}$

ANSWERS

$(0,0)$ $\left(\frac{4}{9}, -\frac{16}{81}\right)$

7) $y = -2x^2 + 2$
$y = 2x^4 - 4x^2 + 2$

Set equations EQUAL TO Each other

$2x^4 - 4x^2 + 2 = -2x^2 + 2$
$+2x^2 - 2 \qquad +2x^2 - 2$

$2x^4 - 2x^2 = 0$

$2x^2(x^2 - 1) = 0$

$2x^2(x+1)(x-1) = 0$

$2x^2 = 0 \qquad x+1=0 \qquad x-1=0$

$\frac{2x^2}{2} = \frac{0}{2}$

$x^2 = 0$
$x = \pm\sqrt{0}$
$x = 0 \qquad x = -1 \qquad x = 1$

Find a y FOR Each X

$X = 0 \quad y = 2(0)^4 - 4(0)^2 + 2 \quad (0,2)$
$y = 2$

$X = -1 \quad y = 2(-1)^4 - 4(-1)^2 + 2 \quad (-1,0)$
$y = 2 - 4 + 2$
$y = 0$

$X = 1 \quad y = 2(1)^4 - 4(1)^2 + 2 \quad (1,0)$
$y = 2 - 4 + 2$
$y = 0$

ANSWERS $(0,2)$ $(-1,0)$ $(1,0)$

9) $3x + y = -4$
$x^2 + y = 0$

Solve top equation for y

$3x + y = -4$

$y = -3x - 4$

Substitute

$x^2 + (-3x - 4) = 0$

$x^2 - 3x - 4 = 0$

$(x + 1)(x - 4) = 0$

$x + 1 = 0$ $\quad$ $x - 4 = 0$

$x = -1$ $\quad$ $x = 4$

Find y's

$x = -1$ $\quad$ $y = -3(-1) - 4$ $\quad$ $(-1, -1)$

$y = 3 - 4$

$y = -1$

$x = 4$ $\quad$ $y = -3(4) - 4$ $\quad$ $(4, -16)$

$y = -12 - 4$

$y = -16$

ANSWERS

$(-1, -1)$ $\quad$ $(4, -16)$

11) $x^2 - y = 0$
$x - y = 0$

Solve bottom EQ. for X

$X - y = 0$

$X = y$

Substitute

$X^2 - y = 0$

$X^2 - X = 0$

$X(X-1) = 0$

$X = 0$ $\quad$ $X - 1 = 0$

$X = 1$

Find y's

$X = 0$ $\quad$ $y = X$ $\quad$ $(0,0)$

$y = 0$

$X = 1$ $\quad$ $y = X$

$y = 1$ $\quad$ $(1,1)$

Answers

$(0,0)$ $(1,1)$

13) $y = -x$
$y = x^3 + 4x^2 + 2x$

Set equations EQUAL to each other

$$x^3 + 4x^2 + 2x = -x$$
$$+x \quad +x$$

$$x^3 + 4x^2 + 3x = 0$$
$$x(x^2 + 4x + 3) = 0$$
$$x(x+1)(x+3) = 0$$

$x = 0$ $\quad x+1=0$ $\quad x+3=0$
$\quad x=-1$ $\quad x=-3$

Find y's

$x = 0$ $\quad y = -x$ $\quad (0,0)$
$\quad y = -0$
$\quad y = 0$

$x = -1$ $\quad y = -(-1)$ $\quad (-1,1)$
$\quad y = 1$

$x = -3$ $\quad y = -(-3)$ $\quad (-3,3)$
$\quad y = 3$

ANSWERS
$(0,0)$ $(-1,1)$ $(-3,3)$

15) $x - 7y + 6 = 12$
$x^2 - y^2 = 36$

Solve top EQ. FOR x

$x - 7y + 6 = 12$
$+7y - 6 \quad -6 + 7y$

$x = 7y + 6$

Substitute

$(7y+6)^2 - y^2 = 36$

$(7y+6)(7y+6)$

$49y^2 + 84y + 36 - y^2 = 36$

$48y^2 + 84y + 36 = 36$
$-36 \quad -36$

$\frac{48y^2}{12} + \frac{84y}{12} = \frac{0}{12}$

$4y^2 + 7y = 0$

$y(4y+7) = 0$

$y = 0$

$4y + 7 = 0$
$4y = -7$
$y = -7/4$

ANSWER
$(6,0)\ \left(-\frac{25}{4}, -\frac{7}{4}\right)$

FIND x

$y = 0 \quad x = 7(0) + 6$
$x = 6$
$(6,0)$

$y = -\frac{7}{4}$
$x = 7\left(-\frac{7}{4}\right) + 6$
$x = -\frac{49}{4} + 6$
$x = -\frac{49}{4} + \frac{24}{4}$
$x = -25/4$

17) $x - 7y + 6 = -4$
$x^2 + y^2 = 20$

Solve top for X

$x - 7y + 6 = -4$
$+7y - 6 \quad -6 + 7y$

$x = 7y - 10$

Substitute

$(7y-10)^2 + y^2 = 20$

$(7y-10)(7y-10)$

$49y^2 - 140y + 100 + y^2 = 20$

$50y^2 - 140y + 100 = 20$
$-20 \quad -20$

$\frac{50y^2}{10} - \frac{140y}{10} + \frac{80}{10} = \frac{0}{10}$

$5y^2 - 14y + 8 = 0$

$(5y - 4)(y - 2) = 0$

$5y - 4 = 0 \qquad y - 2 = 0$

$5y = 4 \qquad y = 2$

$y = 4/5$

Find x

$y = 2$
$x = 7y - 10$
$x = 7(2) - 10$
$x = 14 - 10$
$x = 4$ (4,2)

$y = 4/5$
$x = 7(4/5) - 10$
$x = \frac{28}{5} - 10$
$x = \frac{28}{5} - \frac{50}{5}$
$x = -22/5$ $(-22/5, 4/5)$

Answer
(4,2)
$(-22/5, 4/5)$

19) $3x + y = 8$
$x^2 - 2 + y = 6$

Solve top for y

$y = -3x + 8$

Substitute

$x^2 - 2 + (-3x + 8) = 6$

$x^2 - 2 - 3x + 8 = 6$

$x^2 - 3x + 6 = 6$

$-6 \quad -6$

$x^2 - 3x = 0$

$x(x - 3) = 0$

$x = 0$ $\qquad$ $x - 3 = 0$

$x = 3$

Solve For y

$x = 0$ $\quad y = -3x + 8$

$y = -3(0) + 8$ $\qquad (0, 8)$

$y = 8$

$x = 3$ $\quad y = -3(3) + 8$

$y = -9 + 8$ $\qquad (3, -1)$

$y = -1$

Answers (0,8) (3,-1)